Balancing interferometers with slow-light elements

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In this Letter we show that interferometers with unbalanced arm lengths can be balanced using optical elements with appropriate group delays. For matched group delays of the arms, the balanced interferometer becomes insensitive to the frequency noise of the source. For experimental illustration, a ring resonator is employed as a slow-light element to compensate the arm-length mismatch of a Mach–Zehnder interferometer. An arm-length mismatch of 9.4 m is compensated by a ring resonator with a finesse of 70 and a perimeter of 42 cm. © 2011 Optical Society of America

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Measurements with interferometric sensors can be very sensitive to the frequency noise of the source if the arm lengths are not matched. This can happen in many applications where matching the arm lengths physically is not easily achievable. Stabilizing the frequency of the source by locking it to a stable reference can improve the noise performance of the sensor to some extent [1]. To achieve balanced arm lengths optical delay lines or high-index materials can be used if feasible. For the Laser Interferometer Space Antenna (LISA), different methods are proposed to cancel the effects of phase noise of the source including time delay interferometry [2] and locking the laser frequency to the LISA arm [3]. Our approach for dealing with this issue is matching the group delays of the arms. We show that, by using a slow-light element in the shorter arm of a Mach–Zehnder Interferometer (MZI), the group delays in the arms can be matched to achieve a balanced interferometer.

Recently, slow-light elements have been used in different types of interferometric systems. It was demonstrated that an atomic vapor cell, as an element with negative dispersion in a resonator, can broaden the cavity bandwidth while keeping the cavity buildup high [4]. A slow-light element in one arm of an interferometer has been used to improve the spectral sensitivity [5]. It is also demonstrated that a variable slow-light medium can replace the moving arm of a conventional Fourier transform interferometer [6]. Moreover, slow-light elements have been studied for a wide range of applications including optical gyroscopes [7] and optical communication systems [8], and group delay matching techniques have been employed for optical coherence tomography [9].

In this Letter we first show that a slow-light element in the shorter arm of an unbalanced interferometer can make the output insensitive to frequency deviations of the source. This is conceptually different from [5], where, in a similar scheme, it is shown that the spectral sensitivity of a spectroscopic interferometer is proportional to the group index of the slow-light medium. We also propose using a ring resonator as a slow-light element, which is limited to the cavity bandwidth, and derive the resonator parameters required to compensate the arm-length mismatch. In the experiment, we show that the effects of the intentional frequency modulation of the source can be eliminated by this method.

Figure 1 shows a schematic diagram of an MZI that has a lossless dispersive element with length of L_3 in one arm. The output current of the balanced detector is given by

$$I_{\mathrm{out}} = I_0 \cos(k_0 \Delta L - k_3 L_3 + \phi_0), \tag{1}$$

where $k_0 = \omega/c$ is the free-space wavenumber, $\Delta L = L_1 - L_2$ is the arm-length mismatch, $k_3 = \omega n_3(\omega)/c$, and ϕ_0 accounts for other phase differences of the paths.

The sensitivity of the output to the frequency deviation can be calculated by

$$\frac{dI_{\mathrm{out}}}{d\omega} = I_0 \left(-\frac{\Delta L}{c} + L_3 \frac{dk_3}{d\omega} \right) \sin(k_0 \Delta L - k_3 L_3 + \phi_0). \tag{2}$$

This gives two options for making the output insensitive to frequency changes, i.e., $\frac{dI_{\mathrm{out}}}{d\omega} = 0$. The first option is to make the argument of the sine equal to zero, which is equivalent to matching the optical path lengths in the arms for white-light interferometry. The second option is to make the coefficient of the sine equal to zero, which will give us

$$\frac{\Delta L}{c} = L_3 \frac{dk_3}{d\omega} \Rightarrow v_3^{\mathrm{gr}} \equiv \frac{d\omega}{dk_3} = c \frac{L_3}{\Delta L}, \tag{3}$$

where v_3^{gr} is the group velocity of light in the dispersive element. For $\Delta L \gg L_3$, v_3^{gr} has to be slower than the free-space speed of light by a factor of $L_3/\Delta L$ to make the MZI insensitive to the frequency deviations. This condition is the same as matching the group delays in the arms; that is,

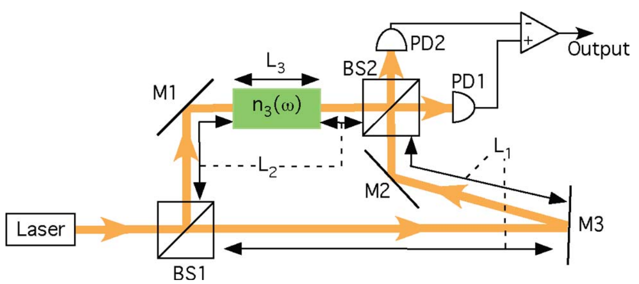

Fig. 1. (Color online) Schematic diagram of an MZI with a dispersive element in one arm. M1, M2, M3, interferometer mirrors; BS1, BS2, beam splitters; PD1, PD2, photodetectors.

$$\frac{L_3}{v_3^{\mathrm{gr}}} = \frac{\Delta L}{c}. \tag{4}$$

One way of realizing a slow group velocity in the shorter arm is using an optical resonator. The required parameters for a lossless three-mirror ring resonator can be calculated using the cavity transmission equation [10]:

$$T_r = \frac{-\sqrt{T_{\mathrm{in}} T_{\mathrm{out}}} \exp(-j\omega L_{\mathrm{in/out}}/c)}{1 - \sqrt{R_{\mathrm{in}} R_{\mathrm{out}} R_3} \exp(-j\omega p_m/c)}, \tag{5}$$

where T_{in}, T_{out} and R_{in}, R_{out}, R_3 are power transmissions and reflections of the mirrors, respectively; $L_{\mathrm{in/out}}$ is the distance between the input and the output mirrors; and p_m is the perimeter of the cavity. Hence the phase response is

$$\angle T_r = \pi - \frac{\omega L_{\mathrm{in/out}}}{c} - \tan^{-1}\left(\frac{g_{\mathrm{rt}} \sin(\omega p_m/c)}{1 - g_{\mathrm{rt}} \cos(\omega p_m/c)}\right), \tag{6}$$

where $g_{\mathrm{rt}} = \sqrt{R_{\mathrm{in}} R_{\mathrm{out}} R_3}$, and $1 - g_{\mathrm{rt}}$ is the round-trip field loss. At resonant frequencies, $\omega_q = \frac{2\pi c q}{p_m}$ (q is an integer), the group delay of the cavity is given by

$$t_{\mathrm{gr}} = -\frac{d}{d\omega} \angle T_r|_{\omega=\omega_q} = \frac{L_{\mathrm{in/out}}}{c} + \frac{p_m}{c}\left(\frac{g_{\mathrm{rt}}}{1 - g_{\mathrm{rt}}}\right). \tag{7}$$

Assuming that $\Delta L = L_1 - L_2 - L_{\mathrm{in/out}}$, to have equal group delays in the arms, the g_{rt} should satisfy the following condition:

$$g_{\mathrm{rt}} = \frac{\Delta L}{p_m + \Delta L}. \tag{8}$$

For $\Delta L \gg p_m$, the group delay matching condition in terms of the cavity finesse is

$$\mathcal{F} \simeq \pi\left(\frac{\Delta L}{p_m}\right). \tag{9}$$

This condition can also be expressed in terms of the cavity storage time and effective path length. For a high-finesse cavity, the storage time and the effective path length are given by

$$\tau_s = \frac{\mathcal{F} p_m}{\pi c}, \qquad L_{\mathrm{eff}} = \frac{\mathcal{F} p_m}{\pi}, \tag{10}$$

respectively [10]. Substitution into Eq. (9) shows that the group delay matching is the same as matching the storage time of the cavity to the time delay difference of the arms. This is equivalent to setting the effective path length of the cavity equal to the arm-length mismatch.

The experimental setup is depicted in Fig. 2. The cavity has a finesse of 70 and a perimeter of 42 cm corresponding to an arm-length mismatch of 9.4 m based on Eq. (9). The laser is locked to the cavity using a Pound–Drever–Hall (PDH) locking system [11], and the interferometer is locked to the mid-fringe using the PZT on M1.

Locking the laser frequency to a cavity resonance mode ensures the appropriate phase response of the cavity where the group delay obeys Eq. (7). On the other hand, frequency locking can suppress the frequency noise of the source over the bandwidth of the controller. This is usually in the kilohertz range, and is set by the frequency response of the actuator. The amount of noise rejection provided by the PDH locking depends on the gain of the controller. Increasing the gain can improve the noise rejection; however, the rejection strength is limited since a very high gain can increase the amount of electronically induced noise and lead to stability issues. In contrast, with our slow-light method, high gain is not required, and one can make the interferometer insensitive to the frequency noise within the transmission band of the cavity, which can be in megahertz or gigahertz range. As the frequency departs from the center of the cavity response, the suppression strength decreases.

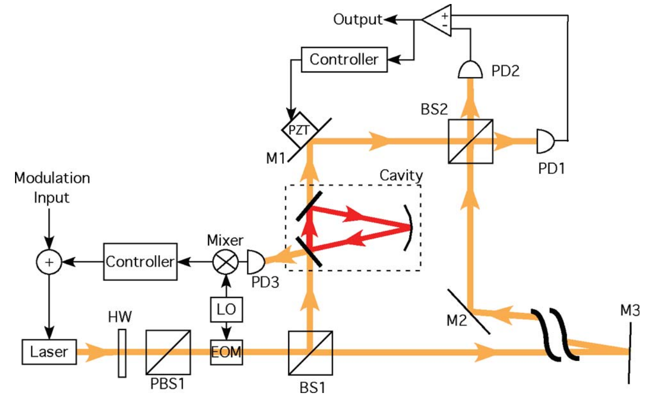

Fig. 2. (Color online) Schematic diagram of an MZI with a resonator in one arm to compensate the arm-length mismatch. Laser frequency is locked to the cavity using a PDH scheme. HW, half-wave plate; PBS1, polarizing beam splitter; EOM, electro-optic modulator; LO, local oscillator; BS1, BS2, beam splitters, PD1, PD2, PD3, photodetectors; M1, M2, M3, interferometer mirrors; PZT, piezoelectric transducer.

A slow-light element, in this case a ring resonator, can be beneficial over the conventional multi-bounce delay lines [12] from the practical point of view. As the arm-length mismatch increases, realizing a delay line becomes more challenging compared to a high-finesse optical resonator on a stable substrate.

One concern in using a resonator as a slow-light element is the effects of mechanical noises, such as thermal expansion of the cavity. A small deviation in the perimeter causes a large deviation in the relative phase of the arms because of the steep phase response at the resonance. A cavity length deviation within the frequency band of the PDH system results in a deviation in the laser frequency. The propagation of frequency deviated light over the distance of ΔL causes the deviation in the relative phases of the arms, σ_ϕ, which is given by

$$\sigma_\phi = \frac{\omega}{c} \frac{\Delta L}{p_m} \sigma_{\mathrm{pm}}, \tag{11}$$

where σ_{pm} is the deviation in the cavity perimeter. On the other hand, the perimeter fluctuations outside the PDH frequency band result in phase changes due to the phase response of the cavity. This phase deviation can be calculated by taking the derivative of the phase response

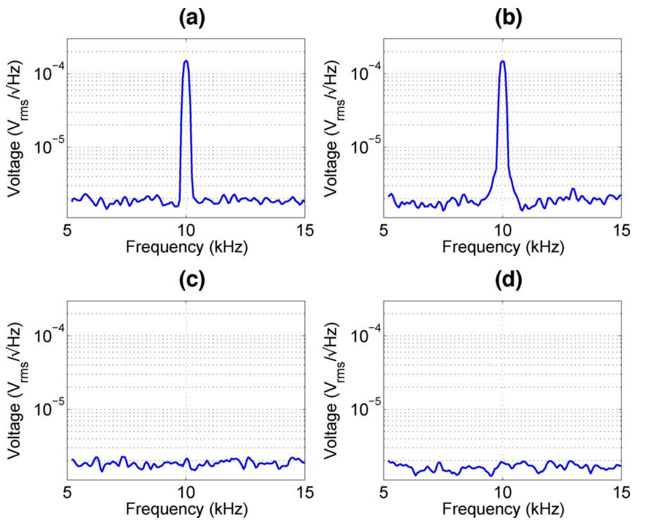

Fig. 3. (Color online) Amplitude spectral density of the MZI output in different configurations while the laser frequency is modulated at 10 kHz: (a) $\Delta L = 9.4$ m without cavity, (b) $\Delta L = 0$ with cavity, (c) $\Delta L = 0$ without cavity, (d) $\Delta L = 9.4$ m with cavity.

[Eq. (6)] with respect to p_m. The result is identical to Eq. (11). It is common to build ring resonators on very stable substrates and minimize these effects. In our experiment, the ring resonator was built on a fused silica substrate, and we did not observe any extra noise due to the mechanical noise of the cavity.

In the experiment, the FWHM transmission bandwidth of the resonator is about 10 MHz, and the PDH controller has a bandwidth of 100 Hz. We intentionally modulated the laser frequency at 10 kHz with a modulation depth of 33 kHz and observed the MZI output in different configurations. 10 kHz is chosen, because there is almost no rejection from the PDH system and it is well within the transmission band of the cavity.

Figure 3 shows the amplitude spectral density of the photodetector output in four configurations. Measurement (a) corresponds to an unbalanced interferometer with arm-length difference of 9.4 m. The frequency modulation of the source caused the large peak at 10 kHz. In (b), the physical arm-length difference is zero, but the cavity exists in one of the arms, i.e., the cavity is used to unbalance the interferometer. The MZI behaves as an unbalanced interferometer, and the peak at 10 kHz has the same magnitude as (a). (c) shows the output of a physically balanced MZI, i.e., $\Delta L = 0$, and (d) is the output of the MZI with 9.4 m arm-length mismatch balanced by the cavity. There is no 10 kHz component above the noise floor in (c) and (d), showing that the ring resonator has balanced the interferometer. By varying the modulation frequency between 1 kHz and 100 kHz (maximum modulation frequency of the laser), the same behavior was observed.

To confirm that the short arm with the cavity behaves as a long arm, we measured the magnitude of the 10 kHz signal for different arm lengths. Results are depicted in

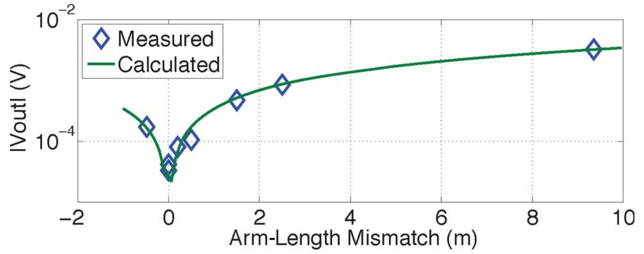

Fig. 4. (Color online) Magnitude of MZI output at 10 kHz versus arm-length mismatch (solid curve is calculated for a simple MZI; y axis is in log scale).

Fig. 4 along with the calculated curve. Zero on the horizontal axis for the measurement corresponds to conditions of Fig. 3(d), and 9.4 m corresponds to Fig. 3(b). For the calculated curve, zero and 9.4 m are equivalent to Figs. 3(c) and 3(a), respectively.

We showed that length mismatch in the arms of interferometers can be compensated with a slow-light element in the shorter arm which can allow white-light interferometry. We experimentally demonstrated the idea by using a ring resonator as the slow-light element. Resonator parameters are calculated, and noise performance is discussed. This new scheme is useful for low-noise interferometric sensors by eliminating the contribution of the frequency noise of the source. For applications that require high sensitivity to the frequency deviations, similar schemes can be used to unbalance the interferometer.

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References

1. J. I. Thorpe, K. Numata, and J. Livas, Opt. Express **16**, 15980 (2008).
2. M. Tinto, F. B. Estabrook, and J. W. Armstrong, Phys. Rev. D **65**, 082003 (2002).
3. K. McKenzie, R. E. Spero, and D. A. Shaddock, Phys. Rev. D **80**, 102003 (2009).
4. G. S. Pati, M. Salit, K. Salit, and M. S. Shahriar, Phys. Rev. Lett. **99**, 133601 (2007).
5. Z. Shi and R. W. Boyd, Opt. Lett. **32**, 915 (2007).
6. Z. Shi and R. W. Boyd, Phys. Rev. Lett. **99**, 240801 (2007).
7. M. Terrel, M. J. F. Digonnet, and S. Fan, Laser Photonics Rev. **3**, 452 (2009).
8. Y. A. Vlasov, M. O'Boyle, H. F. Hamann, and S. J. McNab, Nature **438**, 65 (2005).
9. B. Liu and M. E. Brezinski, J Biomed. Opt. **12**, 044007 (2007).
10. A. E. Siegman, *Lasers* (University Science, 1986).
11. R. W. P. Drever, J. L. Hall, F. V. Kowalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, Appl. Phys. B **31**, 97 (1983).
12. D. R. Herriott and J. Schulte, Appl. Opt. **4**, 883 (1965).